Name:

Fall 2017 Math 245 Final Exam

Please read the following directions:
Please write legibly, with plenty of white space. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may use a single page of notes, but no calculators or other aids. This exam will last 120 minutes; pace yourself accordingly. Please try to keep a quiet test environment for everyone. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
11.	5		10
12.	5		10
13.	5		10
14.	5		10
15.	5		10
16.	5		10
17.	5		10
18.	5		10
19.	5		10
20.	5		10
Total:	100		200

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. even

b. tautology

c. Double Negation theorem

d. Vacuous Proof theorem

Problem 2. Carefully define the following terms:

a. predicate

b. Fibonacci number sequence

c. Proof by Reindexed Induction

d. Proof by Shifted Induction

REMINDER: Use complete sentences.

Problem 3. Carefully define the following terms:

a. recurrence

b. Big Omega

c. $\cup$

d. $=$ (for sets)

Problem 4. Carefully define the following terms:

a. equivalence relation

b. partial order

c. function

d. $=$ (for functions)

Problem 5. For all $x \in \mathbb{R}$, prove that $\lfloor -x \rfloor = -\lceil x \rceil$.

Problem 6. For all $n \in \mathbb{N}$, prove that $\sum_{i=1}^{n} (-1)^i i^2 = \frac{n(n+1)(-1)^n}{2}$.

Problem 7. Find all integers x with $0 \leq x < 24$, that satisfy $9x \equiv 18 \pmod{24}$.

Problem 8. Let A, B, C be sets. Suppose that $f : A \to B$, $g : B \to C$ are surjective functions. Prove that $g \circ f$ is surjective.

Problem 9. For all $x \in \mathbb{R}$, prove that $|x - 1| + |x + 2| \geq 3$.

Problem 10. Let A, B, C be nonempty sets. Suppose that $A \times B = B \times C$. Prove that $A \subseteq C$.

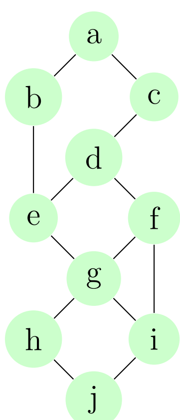
Problem 11. Prove that $n \neq O(\sqrt{n})$.

Problem 12. Let S, T be sets. Carefully state the converse of: If $S \subseteq T$, then $S \cap T = S$. Then, prove or disprove your statement.

Problem 13. Prove that there exists a unique set S, whose full relation is irreflexive.
Note: you must prove both existence and uniqueness for S.

Problem 14. Let $S = \mathbb{Z}$, and $R = \{(a, b) : \max(a, b) \geq 0\}$. Prove or disprove that R is an equivalence relation on S.
Note: $\max(a, b)$ equals the larger of a, b, e.g. $\max(2, 3) = \max(3, 2) = \max(3, 3) = 3$.

Problem 15. Let $S = \mathbb{N}$, and let R be the "divides" relation. Draw the Hasse diagram for the interval poset $[4, 400]$.



The next two problems refer to the poset whose Hasse diagram is pictured.

Problem 16. Find all greatest, maximal, least, minimal, upper bound, lower bound elements, for $T = \{e, f, g\}$.

Problem 17. Find the width and height of the poset pictured above. Prove that your width is correct.

Problem 18. Let $S = \mathbb{Z}$, and $R = \{(a, b) : |a - b| = 2\}$. Find the transitive closure R^+.

Problem 19. Let $S = \mathbb{N}$, and $R = \{(a, b) : a = b + 1\}$. Prove or disprove: R is a function.

Problem 20. Let $S = \{x \in \mathbb{R} : x > 1\}$. Consider the function $f : S \to S$ via $f(x) = \frac{x}{x-1}$. Prove that f is surjective.